

RESEARCH EDITION / OCTOBER 22, 2026

Binary Base-Map Proof Development

Kelly D. Cox | Established results and the remaining global theorem

Edition status. This document presents rigorous local theorems and exact reductions developed in the binary Set, S-M-L, and base-family framework. The available research does **not** yet establish that every positive integer reaches 1. This edition therefore records proof development and its open obligations, rather than claiming a completed proof of the Collatz conjecture.

Every canonical base has a finite path
through actual Collatz family connections to the base of 1.

The boxed statement would complete the connection approach. Numerical descent at every step is unnecessary: a finite valid connection to the rooted component suffices. The central issue is proving that connection for every base.

What this edition establishes and preserves

The first part supplies self-contained proofs of the Set countdown, the Set-1 fall, the unique base-family partition, and the equivalence between global connectivity and Collatz. It also proves an exact inverse-word test and the cofinal inverse-cone characterization. The second part records the later research frontier, including all 22 remaining ledger leaves and the August 21 refinements.

Dates and evidence

October 22, 2026 is the date of this publication edition. Its source manuscripts and scoped reports include research through August 21, 2026. The main frontier's older header date is not treated as the final revision date. Stored computational audits are reported as prior records; the research programs were not rerun to produce this PDF.

Prepared from Kelly D. Cox's research with AI-assisted organization, derivation checks, and typesetting. Internal labels such as 'signed' or 'hostile audit' describe checks in the working archive; they do not denote independent journal peer review. Source identifiers and SHA-256 fingerprints appear at the end.

01 / EXACT STARTING-STATE CLASSIFICATION

Binary Sets and their first descent

Use the stopped ordinary map $C(1)=1$; for $n>1$, let $C(n)=n/2$ when n is even and $C(n)=3n+1$ when n is odd. For odd n , let $A(n)=(3n+1)/2^{v_2(3n+1)}$, the accelerated map that removes all following powers of two. Here $v_2(a)$ is the exponent of 2 dividing a . In particular $A(1)=1$.

Theorem 1. Unique Set coordinates

$$n = 2^s Q - 1 = 2^{s+1} P + 2^s - 1, \\ s = v_2(n+1) \geq 1, Q = 2P+1 \text{ odd}, P \geq 0.$$

Every positive odd integer has exactly one such representation. The Set number s is exactly the number of trailing 1-bits in its binary expansion. Even numbers form Set 0.

Proof. Factor $n+1$ uniquely into 2^s times an odd integer Q . Since n is odd, $s \geq 1$. Writing $Q=2P+1$ gives the second expression. Conversely, that expression has $v_2(n+1)=s$. The binary pattern is $[P] 0 [1 \text{ repeated } s \text{ times}]$, allowing an implicit leading zero when $P=0$. This proves existence and uniqueness.

Theorem 2. The Set-1 fall

Set 1 is precisely $n \equiv 1 \pmod{4}$. It contains half of the odd integers. For $n=4k+1>1$, three ordinary steps give

$$4k+1 \rightarrow 12k+4 \rightarrow 6k+2 \rightarrow 3k+1 < 4k+1.$$

The strict inequality follows from $k \geq 1$. Thus every Set-1 number above 1 genuinely falls below itself within three ordinary steps. For even $n>1$, one ordinary step already decreases n .

Set s has relative density 2^{-s} among odd integers, so Sets 1 through k account for $1-2^{-k}$. These statements count starting integers; they do not assert frequencies along an individual trajectory. Sources: [R1], [R2].

02 / EXACT FINITE TRANSITION

Every higher Set reaches Set 1

Theorem 3. The S-M-L block

Let $S=n=2^sQ-1$, with $s \geq 1$ and Q odd. For $0 \leq j \leq s-1$, its odd checkpoints before the first Set-1 visit are

$$\begin{aligned} n_j &= 3^j 2^{s-j} Q - 1. \\ M = n_{s-1} &= 2 \cdot 3^{s-1} Q - 1. \end{aligned}$$

Proof. If $j < s-1$, then $n_j \equiv 3 \pmod{4}$, so $v_2(3n_j+1)=1$. Applying one odd operation followed by one division gives n_{j+1} . Also $v_2(n_j+1)=s-j$. The Set index drops by one at each such pair of steps. Hence M is in Set 1 after exactly $2(s-1)$ ordinary steps.

For $n > 1$, three more ordinary steps from M reach

$$L = (3M+1)/4 = (3^s Q - 1)/2 < M.$$

This proves finite arrival and a local fall for every higher Set. The root $n=1$ is handled separately by the stopped convention.

The comparison needed for induction

For a higher-Set start $s \geq 2$, the end of this particular block is actually above its start:

$$L - n = ((3^s - 2^{s+1})Q + 1)/2 > 0.$$

For instance, 27 reaches the Set-1 checkpoint 41, whose three-step fall ends at 31. The inequality $31 < 41$ is valid, while $31 > 27$. Additional halving and later blocks may ultimately provide descent below 27; this first block does not.

The Set countdown is already proved. The remaining question concerns the accumulated behavior of successive resets relative to the same original starting value. Sources: [R1], [R2].

03 / INFINITE FAMILIES SETTLED TOGETHER

The unique base-family partition

Define $H(x)=4x+1$ on positive odd integers. Let B be the set of positive odd b with $b \not\equiv 5 \pmod{8}$. These are the canonical bases.

Theorem 4. Unique family and depth

Every positive odd n has exactly one form $n=H^r(b)$,
with $b \in B$ and an integer $r \geq 0$.

Proof. An odd n can be written as $H(x)$ with x positive odd exactly when $n \equiv 5 \pmod{8}$. In that case $x=(n-1)/4 < n$. Repeated removal therefore terminates, and each removal is unique. It yields a unique base b and a unique number r of removed layers. Write $\beta(n)=b$ and $p(n)=r$.

Theorem 5. All family members have the same odd successor

$$\begin{aligned} 3H^r(b)+1 &= 4^r(3b+1), \\ \text{so } A(H^r(b)) &= A(b). \end{aligned}$$

The identity follows by induction on r , and removing powers of two gives the conclusion. Thus proving convergence for one base settles its entire infinite H -family.

Conversely, an odd target y has odd predecessors only if 3 does not divide y . Put $\epsilon(y)=2$ when $y \equiv 1 \pmod{3}$, and $\epsilon(y)=1$ when $y \equiv 2 \pmod{3}$. Then

$$\begin{aligned} b(y) &= (2^{\epsilon(y)}y-1)/3, \\ A^{-1}(\{y\}) &= \{H^r(b(y)) : r \geq 0\}. \end{aligned}$$

Proof. $A(n)=y$ means $3n+1=2^e y$. Modulo 3, the admissible positive exponents are exactly $e=\epsilon(y)+2r$. Substitution gives the displayed family. Its smallest element is canonical because its exponent ϵ is either 1 or 2; removing an H -layer would reduce that exponent by 2. Thus the families are exactly the fibers of A , not merely subsets of them.

Example: 3, 13, 53, ... all have odd successor 5. Since 5 reaches 1, every member of this infinite family does. Sources: [R1], [R2].

04 / WHAT THE BASE-TREE TABLE PROVES

Exact connections between families

For a canonical base b , the next canonical family is determined by

$$\pi(b) = \beta(A(b)).$$

This rule has one outgoing connection from every base. It need not have only one incoming connection. The identity in Theorem 5 gives $\beta(A(n)) = \pi(\beta(n))$; hence the family map preserves the full forward question.

Theorem 6. The quotient loses no convergence information

An odd n reaches 1 under A if and only if $\beta(n)$ reaches 1 under π . The forward implication follows by projecting every step. Conversely, if a projected step lands at base 1, the actual odd state is $H^r(1)$ for some finite r , and its next odd successor is 1 by Theorem 5. The case n already in that family is included.

The table's row and column coordinates

Let $E=3b+1$, an even integer congruent to 4 modulo 6. The row with header E has odd entries $H^r(b)$, and even partners $4^r E$. Its closed form is

$$H^r(b) = 4^r b + (4^r - 1)/3.$$

Base b	Even header E	Odd family	Common odd target
1	4	1, 5, 21, 85, ...	1
3	10	3, 13, 53, 213, ...	5
7	22	7, 29, 117, 469, ...	11

Every positive odd row label $b=(E-1)/3$ occurs as E ranges over 4,10,16,22,... . These labels include canonical bases and higher members of their families. Duplicate higher H -layers reduce to the same canonical family. This is exhaustive placement; it remains to prove that every row eventually joins the row of 1.

The particular header 22 is already rooted: $22 \rightarrow 11 \rightarrow 34 \rightarrow 17 \rightarrow 52 \rightarrow 26 \rightarrow 13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$. This example is settled; the open obligation is universal coverage. Sources: [R1], [R2].

05 / A SUFFICIENT GLOBAL THEOREM

Connectivity would finish the proof

Theorem 7. Root connectivity is equivalent to Collatz

Form the graph whose vertices are canonical bases and whose directed edges are $b \rightarrow \pi(b)$. Ignore arrow directions only when asking whether two vertices are connected. A connection means a finite path. Then

Every positive integer reaches 1
 $\Leftrightarrow$ every canonical base is connected to base 1.

Proof. A forward path is also an undirected path, giving one implication. For the reverse implication, take a shortest undirected path $v_0, \dots, v_k = 1$. Since $\pi(1) = 1$, its last nontrivial edge points toward 1. Moving backward along the simple path, each vertex already has its unique outgoing edge toward the next vertex, so the preceding edge must also point toward 1. The whole path is a forward path. Theorem 6 returns this conclusion to odd integers, and even integers halve to an odd one.

A merger gives a valid induction certificate

More generally, two vertices in a graph with one outgoing edge are connected exactly when their forward orbits meet after finitely many steps. Along one edge the two orbits eventually agree; this property passes along a finite chain. Therefore a sufficient and necessary universal induction statement is

For every $b \in B$ with $b > 1$, there are $c \in B$ with $c < b$
 and integers $p, q \geq 0$ such that $\pi^p(b) = \pi^q(c)$.

Proof of sufficiency. Strongly induct on the numerical base b . By induction c reaches 1; its common future with b then roots b . Necessity follows from choosing $c = 1$ when every base is rooted.

The shared future may be reached while b itself is still above its original size. Thus connectivity or a smaller merging source is a valid route to proof even without a separately proved first numerical descent.

The current archive establishes the graph and its links. It has not established the universal finite-root-path statement. Sources: [R1], [R2].

INDUCTION / EXACT FAMILY CLOSURES

A smaller remaining canonical core

The preceding results give actual induction gains. If $n=H^r(b)$ with $r>0$, then $b<n$ and $A(n)=A(b)$, so the smaller base settles n . Set-1 numbers above 1 descend directly. Therefore a least unresolved odd source would have to be a canonical higher-Set base $b=4x-1$. This is a classified class, not an unknown type of number.

A uniform smaller-source certificate

If $x=3k$, let $b=12k-1$ and $c=8k-1$. Then c is positive and canonical, $c<b$, and

$$3c+1 = 2b, \text{ so } A(c)=b \text{ and } \pi(c)=b.$$

Thus the convergence of c implies the convergence of b . This certifies every base in this residue class by strong induction, even though b 's own next odd step increases it.

The recorded two-certificate split

Write $x=2^a Q$ with Q odd. The detailed induction report [R11] establishes a second finite merger certificate when $Q \equiv 3^a \pmod{4}$. Together, the two sufficient conditions are

$$3 \text{ divides } x, \text{ or } Q \equiv 3^a \pmod{4}.$$

Their union has relative density $2/3$ within the canonical higher-Set bases. The exact residual is 3 not dividing x and $Q \not\equiv 3^{a+1} \pmod{4}$, beginning at base 7. The fraction records the scope of these particular certificates; it is not a claim that only $1/3$ of integers remain unverified.

Further exact return and inverse-sibling tests settle additional proper subclasses, with comparisons always made to the same original base. The later V6 ledger records the remaining guarded obligations. A residual class can contain many numbers already known to reach 1; 'open' means the specified certificates do not establish the entire class at once.

Sources: [R1], [R2], [R11]. Only the first smaller-source certificate is proved in full on this page; the second is reported at its archived scope.

06 / EXACT, FINITE INVERSE CERTIFICATES

A complete test for an inverse word

Start with a positive odd target t . Choose a finite word $f_1, \dots, f_q$ of positive integers, interpreted outward from that target. Define $F_j = f_1 + \dots + f_j$, $D_0 = 0$, and

$$\begin{aligned} D_j &= 2^{f_j} D_{j-1} + 3^{j-1}, \\ y_j &= (2^{F_j} t - D_j) / 3^j. \end{aligned}$$

Theorem 8. Endpoint congruence suffices for a literal word

For this exact recurrence, if $2^{F_q} t \equiv D_q \pmod{3^q}$, every intermediate y_j is a positive odd integer. Moreover $A(y_j) = y_{j-1}$, with the exact exponent f_j .

Proof. Expanding D_q shows $D_q \equiv 2^{F_q - F_j} D_j \pmod{3^j}$. The terminal congruence, and invertibility of 2 modulo every power of 3, force $2^{F_j} t \equiv D_j \pmod{3^j}$. Thus all prefixes are integral. The recurrence is $y_j = (2^{f_j} y_{j-1} - 1) / 3$. Starting from $t > 0$ odd, its integral value is positive and odd. Finally $3y_j + 1 = 2^{f_j} y_{j-1}$ gives the exact valuation.

Corollary. A smaller source below the original floor

For an original positive odd $m > 1$, the word yields $0 < y_q < m$ exactly when its terminal congruence holds and

$$3^q \leq 2^{F_q} t - D_q \leq (m-2)3^q.$$

The upper endpoint uses the fact that y_q and m are odd. If t is an actual future checkpoint of m , this is a smaller merging-source certificate for m .

Scope correction. This derivation sharpens the warning in [R4]: internal congruences follow from the terminal one for the literal carry recurrence above. They do not follow for arbitrary relaxed or projected affine variables. The test verifies a proposed word; it does not prove that a successful word exists for every m .

07 / ARBITRARY FUTURE CHECKPOINTS

The cofinal inverse-cone theorem

For odd m , define its merge component $C(m)$ as the positive odd z whose accelerated orbit meets the orbit of m . Let $P(t)$ contain t and all positive odd finite inverse predecessors of t . Choose any unbounded sequence of nonnegative times t_i and put $b_i = A^{t_i}(m)$.

Theorem 9. Cofinal checkpoints cover the whole component

$$C(m) = \bigcup_i P(b_i).$$

Proof. Every inverse predecessor of a checkpoint merges with m , giving one inclusion. If $A^p(z) = A^q(m)$, choose $t_i \geq q$. Then $A^{p+t_i-q}(z) = b_i$, so z belongs to the inverse cone of that checkpoint. This proves the reverse inclusion.

Consequently m is the minimum of its merge component precisely when every positive inverse endpoint at every chosen cofinal checkpoint is at least m . Combining this fact with Theorem 8 makes the missing witness completely explicit.

Find a future checkpoint of m and a finite literal inverse word whose endpoint is a positive odd integer smaller than m .

What is proved, and what is not supplied

The theorem permits arbitrary future time and arbitrary finite inverse depth. It also allows use of a sparse sequence of actual checkpoints, provided their times are unbounded. It proves that such checkpoints lose no possible merging source.

It does not force a smaller member to exist. If m were a component minimum outside the root component, the exact identity would still hold, with every inverse endpoint at least m . Ruling out that situation is the global step.

The induction target can therefore be expressed as: every positive odd $m > 1$ has some smaller positive member of $C(m)$. This is equivalent to Collatz by strong induction on m . The archive further reduces the target to its terminal representatives; that reduction preserves the same original-source comparison. Sources: [R1], [R4].

08 / WHY FIXED-DEPTH ARGUMENTS STOP

Finite runs can grow through every reset

Local falls and finite Set countdowns do not provide a uniform number of blocks by which every start must descend. The following explicit family establishes that limitation without constructing an infinite counterexample.

Theorem 10. Arbitrarily long finite rising block sequences

Choose $K \geq 1$ and any positive multiple q of 3. For $0 \leq j \leq K$, define

$$\begin{aligned} d_j &= q \cdot 9^j 2^{3(K-j)+3} - 5, \\ M_j &= (3d_j + 1)/2. \end{aligned}$$

For $j < K$, $d_j \equiv 3 \pmod{8}$. Its exact ordinary path is

$$\begin{aligned} d_j &\rightarrow 2M_j \rightarrow M_j \rightarrow 4d_{j+1} \\ &\rightarrow 2d_{j+1} \rightarrow d_{j+1}. \end{aligned}$$

Proof. Direct substitution gives $d_{j+1} + 5 = (9/8)(d_j + 5)$ and $3M_j + 1 = 4d_{j+1}$. The displayed congruence ensures that d_j has exactly one initial division and M_j is Set 1. The terminal d_{j+1} is odd. Moreover

$$\begin{aligned} d_{j+1} - d_j &= (d_j + 5)/8 > 0, \\ M_j - d_{j+1} &= (3d_j - 1)/8 > 0. \end{aligned}$$

Each block contains a genuine Set-1 fall, but its endpoint exceeds the preceding block's start. All displayed states remain at or above the original d_0 . For $K=1$ and $q=3$, the example is $187 \rightarrow 562 \rightarrow 281 \rightarrow 844 \rightarrow 422 \rightarrow 211$.

For every requested finite K there is such a starting value. The value changes with K . This does not yield one positive integer that rises forever, and does not exclude an input-dependent induction or connectivity argument. Source: [R2], finite-horizon obstruction and lossless-state discussion.

09 / CURRENT PROOF BOOKKEEPING

The frozen V6 frontier

The coherent V6 coverage ledger is an exhaustive partition of its stated terminal-quotient cases. Its recorded status is unchanged by the later results summarized here. [R1], [R3]

Ledger quantity	Recorded count	Meaning
Nodes	92	Cases and intermediate containers
Exact partitions	31	Exhaustive case splits
CLOSED nodes	39	Scoped descent/merger or impossibility established
OPEN nodes	53	Includes containers as well as unresolved leaves
OPEN leaves	22	Remaining terminal obligations

Open branch family	Leaf count
O_C3	16
O_HIGH	2
O_T	1
O_D	3

The 22 leaves are not 22 numbers or counterexamples. Each is a precisely guarded class of candidate behavior. Nor is 39/92 a percentage of Collatz solved: node counts depend on how the proof is organized.

V6 adds five proper subclass closures through four theorem packages: a C3 sharp-height condition; an eventual contraction cap applied to two high branches; an eventual low hidden-depth tail; and a D-return component-floor condition. Each leaves an exact open complement. No entire predecessor open leaf was promoted to closed.

A recorded periodic exclusion applies to that periodic sublanguage only unless it also yields a smaller source for an occurrence inside an arbitrary orbit. The appendix lists the exact current OPEN leaf identifiers so this summary can be matched to the machine-readable ledger.

10 / LATER ALL-DEPTH RESEARCH

What the refined arguments accomplish

Geodesicity and merger depth

The archive proves an all-length comparison for exponent words in $\{1,2\}$ with no adjacent twos: among compatible words of the same length and the same ternary target, the source word has minimum total exponent, uniquely at equality. The stronger gap-five application shows that a smaller positive merger into a rising legal source must have greater odd-step depth. This is a restriction on possible certificates, not their universal exclusion. [R1]

A concrete greater-depth merger remains possible: 1471 reaches 8383 by exponent word 111112 (six odd steps), while 1307 reaches the same target by 12111112 (eight steps). Thus deeper comparisons cannot simply be removed from the proof problem.

Formal-zero exclusions and finite protection

Within the stated residual macro-boundary domain, the all-sign formal-zero arguments exclude the specified contracting affine comparisons. Separate theorems protect arbitrary prescribed finite merger horizons for infinitely many sources and construct longer finite zero-append runs. The sources depend on the chosen horizon or run length. [R1]

A complementary Hensel construction places an explicit smaller merger beyond any prescribed finite protection horizon. Its deeper merger depth is an upper bound on the first such depth, not necessarily the first. These results show that no fixed finite inspection horizon resolves all sources; they do not settle a fixed source at arbitrary depth. [R1]

The ordinary-integer requirement

Finite realizability, real affine inequalities, rational local models, and 2-adic limits must all be distinguished from one coherent positive ordinary integer satisfying the entire infinite schedule. The refined work repeatedly identifies this as the remaining arithmetic requirement.

These are scoped symbolic theorems and recorded audits from the working archive. Their domains and quantifiers are essential: no arbitrary-depth merger exclusion, complete open-leaf closure, global cycle exclusion, or Collatz conclusion is asserted here.

11 / A CRITICAL INFINITE FORMAL SCHEDULE

Finite prefixes versus one fixed integer

The terminal H2 study constructs an infinite formal schedule whose terminal Set labels are only 1 and 2, whose hidden ternary depths are zero, and whose resets are one. Its prefixes have the exact critical odd density behavior and high symbolic complexity described in [R5].

The proved quantifier

For every finite K , there are infinitely many positive integers realizing the first K rows of this schedule.

Each such finite realization remains above its own source throughout the shared prefix. The nested binary congruence classes determine one 2-adic center. The report does not establish that this center is a positive ordinary integer, nor does it establish that it is not.

Why this matters to the binary argument

All ordinary positive integers have finite binary expansions. A compatible infinite family of congruences can instead define a 2-adic value outside those positive integers. Therefore an arbitrary supply of finite examples cannot be exchanged for a single positive starting integer satisfying every row.

For every K there exists a source d_K
does not imply there exists one source d for every K .

This construction does not establish a divergent positive Collatz orbit or a counterexample. It demonstrates why the local Set rules, finite-cylinder realizability, and critical-density calculations do not by themselves settle the fixed-integer question.

A precise remaining target

A successful binary completion could prove that every positive ordinary center compatible with an indefinitely surviving schedule must eventually yield a smaller source or a finite connection to the root. The archive contains substantial exclusions of restricted schedules, but not that universal theorem.

The H2 notation here concerns the terminal Set bound in the research report; it is distinct from the family function $H(x)=4x+1$ used earlier. Source: [R5].

12 / RESEARCH THROUGH AUGUST 21

Switch arithmetic and rational models

Fixed-source residual refinement

For residual macro depths $u_n \in \{6, 7\}$, the exact core recurrence is $3^{u_n}c_n + 1 = 2^{u_{n+1}+1}c_{n+1}$. Put $X_n = 2^{u_n}c_n$. Then

$$X_{n+1} = (3^{u_n}/2^{u_n+1})X_n + 1/2 > X_n.$$

The August 21 refinement derives weighted complexity constraints, lower bounds on switching, constraints on growing prime support, and exact gcd relations for a hypothetical fixed positive residual chain. It also gives finite CRT families defeating proposed monotonicity arguments for normalized units. Its independent audit records a clean result within that scope. [R6], [R7]

The earlier switch equations have the form $1931 \cdot 3^{6r}a - 1 = 601 \cdot 2^{8s}b$ and $601 \cdot 3^{7s}b + 1 = 1931 \cdot 2^{7r'}a'$. The recorded S-unit bound forces infinitely many different primes along an infinite fixed-source switch chain. Changing prime support remains possible under the recurrence; the bound is therefore a necessary condition, not a contradiction.

Rational local models and their limits

An independently audited O_T v2 construction gives a positive rational cycle satisfying its localized row predicates. Its reduced denominator prevents it from being a positive ordinary-integer orbit. The older predecessor is explicitly void and is not evidence in this edition. [R8]

An independently audited O_D construction has the same limitation: an exact rational cycle satisfies its local predicates, but its nonunit denominator excludes an ordinary-integer orbit. [R10]

A separate August 21 O_C3 note presents a rational local model and explicitly requires a fresh independent audit. It is included here only as a pending research item, not a new certified leaf result. [R9]

These models show the importance of retaining ordinary integrality when using relaxed equations. They are not Collatz counterexamples, do not demonstrate inhabitants of the ordinary open leaves, and do not change the V6 status.

The October publication date does not re-date these August results. This edition preserves their recorded scope and distinguishes independently checked notes from pending scratch work.

13 / THE PRECISE REMAINING THEOREM

How the approach can be completed

The proved family and component reductions leave a concrete mathematical target:

For every positive odd $m > 1$, there exists a positive odd $z < m$
whose forward orbit meets the forward orbit of m .

Theorem 7 shows that a canonical-base version suffices, and the terminal reduction in the archive further narrows the representatives. Theorem 9 permits the witness to be sought at any actual cofinal checkpoint. Theorem 8 gives a complete finite arithmetic test for each proposed literal inverse word.

Two coefficient branches remain

At a finite first coefficient crossing, the recorded lift reduction settles noncanonical lifts, while a terminal canonical base can remain unless it has a later value or a smaller merging source below the same original base. At infinite coefficient time, the open issue is excluding a divergent component minimum. The same smaller-component-source theorem subsumes both branches. [R1]

What would be sufficient

A proof of universal finite root connectivity would suffice directly. A proof of the smaller merging-source statement would suffice by strong induction. A well-founded rank on the exact unbounded state, proved to decrease until the root, would also suffice. None of these needs a uniform bound on the number of steps for all inputs.

What the current work does not conclude

Exhaustive placement, all local edge formulas, finite Set countdowns, numerous exact family closures, and the absence of counterexamples in finite audits do not yet establish the required global theorem. Conversely, the open leaves and formal survivors are not demonstrations that the framework cannot succeed.

The established theorems give an exact representation of the problem and several rigorous reductions. This publication's conclusion is that global root connectivity remains open in the available argument. No completed proof or disproof of Collatz is claimed.

APPENDIX A / LEDGER IDENTIFIERS

The 22 remaining open leaves

These identifiers are reproduced from [R3]. Their full predicates, inherited precedence conditions, and evidence links remain in the machine-readable ledger. The labels alone are not substitute definitions.

No.	Exact OPEN leaf identifier
1	C3_ONE_W_C3_RESET1_EXPANDING_OPEN
2	C3_ONE_W_D3_EXPANDING_OPEN
3	C3_ONE_W_ORDINARY_HIGH_EXPANDING_OPEN
4	C3_ONE_W_HIDDEN_EXPANDING_OPEN
5	D_RESETONE_FIRST_CONTRACTION_BAND_EQUALITY_OPEN
6	D_RETURN_U_AFTER_FIRST_CONTRACTION_BAND_RESIDUAL_OPEN
7	C3_ZERO_W_Z2_W_OPEN
8	C3_ZERO_W_U3_Z2_H_OPEN
9	C3_ZERO_W_U3_W_D1_OPEN
10	C3_ZERO_W_U3_W_W_OPEN
11	C3_ZERO_W_U3_W_H_OPEN
12	C3_ZERO_W_V3_KGE2_HIGH_RESET_EXIT_OPEN
13	C3_ZERO_W_V3_U1_D1_OPEN
14	C3_ZERO_W_V3_U1_W_OPEN
15	C3_ZERO_W_V3_U1_H_OPEN
16	C3_ZERO_W_U3_D1_LOW_RESET_TAILS_OPEN
17	C3_ZERO_W_Z2_D1_LOW_RESET_TAILS_OPEN
18	C3_ZERO_W_V3_Z2_BELOW_H_OPEN
19	HIGH_INITIAL_COFINAL_RECHARGE_OPEN
20	HIGH_AFTER_PREFIX_COFINAL_RECHARGE_OPEN
21	T_CRITICAL_NO_EVENTUAL_LOW_HIDDEN_TAIL_OPEN
22	D_RETURN_Z_DLEH_OPEN

APPENDIX B / SOURCE RECORD

Research provenance and audit scope

The references below are internal research artifacts, not claims of external peer review. The core proofs are reproduced in this PDF; advanced results are summarized with their recorded restrictions. A separate machine-readable source manifest accompanies the local build.

[R1] Main proof frontier

binary-base-map-proof-frontier.md

[R2] Exact global analysis

binary-base-map-global-analysis.md

[R3] V6 coverage ledger

tmp/global-grammar-coverage-v6-four-refinement/coverage-tree.json

[R4] Cofinal inverse component reduction

tmp/cofinal-inverse-component-reduction/cofinal-inverse-component-reduction.md

[R5] Critical H2 formal survivor

tmp/terminal-h2-critical-survivor/terminal-h2-critical-survivor.md

[R6] Fixed-source residual refinement

tmp/residual-fixed-source-global-refinement/residual-fixed-source-global-refinement.md

[R7] Independent residual refinement audit

tmp/residual-fixed-source-global-refinement-hostile/HOSTILE_AUDIT.md

[R8] Rational O_T v2 independent audit

tmp/v6-ot-rational-countermodel-v2-hostile/HOSTILE_AUDIT.md

[R9] Rational O_C3 scoped scratch note

tmp/v6-oc3-one-w-c3-rational-countermodel/v6-oc3-one-w-c3-rational-countermodel.md

[R10] Rational O_D independent audit

tmp/v6-od-rational-countermodel-hostile/HOSTILE_AUDIT.md

[R11] Canonical higher-Set induction split

tmp/canonical-higher-set-induction-split/canonical-higher-set-induction-split.md

The main frontier and global-analysis manuscripts retain the complete derivations, links to scoped reports, and prior computation logs. Later scratch notes are not silently promoted to theorems. This edition's inverse-word lemma explicitly records the scope correction to [R4].

APPENDIX C / IMMUTABLE SOURCE FINGERPRINTS

Snapshot used for this edition

SHA-256 fingerprints identify the exact files read for this edition. They record provenance, not a proof of mathematical correctness. Dates are stated in the source documents; filesystem modification times were used only to locate the latest work.

[R1] Main proof frontier

042c6b5f05aba40d1219db3ed0227cf23779a49492b4a06b1d4d5bcd0aff391d

[R2] Exact global analysis

14c9cae871ac8ed843fa7e827bbc711f91daca13ea8cf51732354c7aca8fed3a

[R3] V6 coverage ledger

9a5a0fe86a7223c110b7a43a1fb0840e23822605cd8b69a7d26f1224b157d690

[R4] Cofinal inverse component reduction

97099e241b6df986413d1805d346f06ad9ac9f03814e83bab5c8c733b123f95c

[R5] Critical H2 formal survivor

3e904fce39a940a04dc311cc9a3c58a300f4c3b9d10bc00af18d5840a0c112b5

[R6] Fixed-source residual refinement

fcad53695161f9444caaab5b3e62e55e0211be2c70b0bcb367f0e7479aa9c78

[R7] Independent residual refinement audit

394cacc38b44df230e649c05d8a1d9dfb6e2eee47de6b0ca0b35d8abde2172f9

[R8] Rational O_T v2 independent audit

103b01351ee91465df654b1fb20bef2c62b2059cefbb17e026918f9af350a5ea

[R9] Rational O_C3 scoped scratch note

15f839b6c0bb7a8359e5fedeed8fe6881fa1dac607094787558ae870b12bbce3

[R10] Rational O_D independent audit

625690de0cf57c61aa720cab042458223eeb2213cf9593355272bf717e2a675f

[R11] Canonical higher-Set induction split

2d974b886cdeaaa349257634072271df94b71a80835eb3925dc9efddfccbbab

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